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Class: -S.Y.B.Sc. C.S.	Subject: - Linear Algebra
Semester:- III	Course code: -USCS302
Exam Event:- Additional Exam Summer 2024 (FH)	Marks: -75
Date: - 23/03/2024	Duration: - 02:30 Hours

- N.B. –**
- 1 – All questions are compulsory.
 - 2 – All questions have internal choice.
 - 3 – Figures to the right indicate full marks.

Q1. Answer the following question. (Any 4 out of 6)

20

- 1) Prove or give counter example for the statement “ $\{[X,Y,Z] | X,Y,Z \in \mathbb{R}, X+Y+Z=1\}$ is a vector space”?
- 2) Solve the system of linear equation:
 $2x_1 - 4x_2 + 9x_3 = 7; 5x_2 - 2x_3 = 2; 12x_3 = 5;$
- 3) Write a python program to rotate a complex number by 90° , 180° and 270° with output.
- 4) Given vectors $u=[0,1,1]$ and $v=[1,1,1]$ over $GF(2)$, find the following vectors:
a) $v+u$ b) $v+u+u$ c) $u+v+v$
- 5) Find a vector $x=[x_1, x_2, x_3, x_4]$ over $GF(2)$ satisfying the following linear equations:
 $1100.x=1; 1010.x=1; 1111.x=1;$
- 6) Calculate the dot product $u \cdot v$ for the following pairs of vectors over \mathbb{R} :
a) $u=[1,0], v=[5,16,16]$ b) $u=[1,2,3], v=[3,2,1]$
c) $u=[-\sqrt{2}/2, \sqrt{2}/2], v=[\sqrt{2}/2, -\sqrt{2}/2]$ d) $u=[15,30, -2], v=[-12,19,19]$

Q2. Answer the following question. (Any 4 out of 6)

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- 1) Find the null space of matrix $A = \begin{bmatrix} 1 & 5 & 6 \\ 2 & 6 & 8 \\ 3 & 4 & 7 \end{bmatrix}$
- 2) Test the linear dependency and find the relationship if it exist for $X_1=(1,2,3)$, $X_2=(3,-2,1)$, $X_3=(1,-6,5)$
- 3) Given $V = \text{Span} \{[0,1,0,1], [0,0,1,0], [1,0,0,1], [1,1,1,1]\}$ are over $GF(2)$, for the vector $[0,1,1,1]$. Show it belong to v by expressing it as a linear combination of generators of V .
- 4) Define coordinate system. Find the coordinate representation of $V=[1,3,5,3]$ in terms of $a_1=\{1,1,0,0\}$, $a_2=\{0,1,1,0\}$ and $a_3=\{0,0,1,1\}$
- 5) Given $V = \text{Span} \{v_1=[2,0,4,0], v_2=[0,1,0,1], v_3=[0,0,-1,-1]\}$, for the vector $[0,1,1,2]$. Show it belongs to v by expressing it as a linear combination of generators of V .
- 6) Define coordinate system. Find the coordinate representation of $V=[0,0,0,1]$ in terms of $a_1=\{1,1,0,1\}$, $a_2=\{0,1,0,1\}$ and $a_3=\{1,1,0,0\}$ over $GF(2)$.

Q3. Answer the following question. (Any 4 out of 6)

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- 1) Given vector $v_1=[1,1,1,1]$, $v_2=[1,2,4,5]$ and $v_3=[1, -3,-4,-2]$, construct an orthogonal set of generators for sub space of \mathbb{R}^4 whose generators are v_1, v_2, v_3 .
- 2) Determine a basis for W^\perp where $w = \text{Span} \left(\begin{bmatrix} 1 \\ -5 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right)$

3) Define characteristic polynomial of degree 2 matrix. Find the characteristic

polynomial of matrix $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

4) Find a basis for the row space of matrix A and determine the dimension of the row space for

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} 0$$

5) Apply the Cayley-Hamilton theorem to find the eigenvectors of the matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

6) Explain the concept of orthogonality in linear algebra.

Q4. Answer the following question. (Any 5 out of 6)

15

- 1) Given point P= [3,2] and Q= [2,4], calculate \overrightarrow{PQ} and \overrightarrow{QP} .
- 2) Explain the dictionary-based representation of vectors.
- 3) What is the kernel of a linear transformation?
- 4) Write Python code for both inner and outer product of vectors.
- 5) Briefly describe the concept of Google Page Rank.
- 6) Discuss internet worm and how does it function.